# Ampère Tension in Electric Conductors

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**Abstract:** It is shown that the application of the Ampère and Lorentz force laws to a closed current in a metallic circuit results in two different mechanical force distributions around the circuit. In addition to the transverse forces, which both laws predict, the Ampère electrodynamics requires a set of longitudinal forces that subject the conductor to tension. These longitudinal forces explain electromagnetic jet propulsion and the recoil mechanism in a railgun. Pulse current experiments are described in which Ampère tension shattered solid aluminum wires. Electrons moving through the metal lattice are the basic current-elements of the Lorentz force theory. But Ampère assumed his current-elements to be infinitely divisible. With the help of computer-aided analysis and experiment, it is demonstrated that the amperian current-element must also be of finite size and involve at least one lattice ion in addition to the conduction electron. Calculations with Ampère's formula have been found to give reasonable results when the atom, or unit atomic cell, is taken to be the smallest possible current-element. Some technological consequences of Ampère tension are discussed briefly with regard to pulse currents in normal conductors and steady currents in superconductors. The use of large macroscopic current-elements of unit length-to-width ratio gives rough approximations to the Ampère tension. The accuracy of the calculations can be improved by resolving the conductor into a number of parallel filaments, each filament being subdivided into cubic current-elements.

## INTRODUCTION

### A. Historical

In the 160 years which have elapsed since Oersted's discovery of electromagnetism,[1] Ampère's force law[2] reigned supremely during the first 80 years. Maxwell[3] said:

> It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics.

This statement of Maxwell's has lost little of its validity during the past 100 years provided the term phenomena is restricted to mean electric-current phenomena in metallic conductors. Lorentz[4] found it necessary to substitute the transverse force known by his name for amperian repulsions and attractions, primarily to explain the motion of electrons in vacuum. He recognized that this might cause conflict with Newton's third law of motion. By now generations of physicists have unhesitatingly assumed that the magnetic force acting on a moving electron in vacuum remains unchanged when this electron travels through the metal lattice.

Ampère's force law formed the basis of F.E. Neumann's mathematical theory of electromagnetic induction.[5] The resulting Ampère-Neumann electrodynamics, as taught in the nineteenth century, embraced much indirect experimental evidence of the existence of mechanical forces of electromagnetic origin which acted parallel to the current streamlines. The literature of the time also contains two direct demonstrations of longitudinal forces.[6,7] The author's discovery of electromagnetic jet propulsion[8] and Hering's[9] liquid metal pumps represent further demonstrations of longitudinal Ampère forces.

### B. The Force Laws

In the years from 1820 to 1825 Ampère carried out many experiments concerning the mechanical forces exerted between current-carrying metallic conductors. Eventually he singled out four null experiments,[2] that is, experiments in which the forces balanced each other, from which he deduced his fundamental force law for the interaction $\Delta F_{m,n}$ of two current-elements $i_m\,dm$ and $i_n\,dn$. Of the many forms in which his formula may be written, perhaps the most useful is

$$\Delta F_{m,n} = -i_m i_n\,(dm \cdot dn/r^2_{m,n})\,(2\cos\varepsilon - 3\cos\alpha\cos\beta) \qquad (1)$$

where $dm$ and $dn$ are the lengths of the two elements and $r_{m,n}$ is the distance between their center points. Figure 1 depicts the three angles of Ampère's force law. The inclination of the current-element vectors is defined by $\varepsilon$, and $\alpha$ and $\beta$ are the inclinations of the current elements to the distance vector $r_{m,n}$. As all three angles appear in cosines, and since the cosine is the same for positive and negative angles, it does not matter in which direction the vectors are turned to bring them into coincidence. Furthermore, of the two possible orientations of the distance vector, either will give the same interaction force. It helps to imagine that the directional properties of the current-element are vested in the element length and not in the current.

Equation (1) has been treated as the defining equation of fundamental electromagnetic units. If the currents are inserted in absolute ampere (1 ab-amp = 10 A), the force is given in dyne.

The force law for two current-elements which has been used during the past 80 years was first proposed in 1845 by Grassmann,[10] who also invented the vector calculus. It is an unsymmetrical law and therefore has to be stated by two

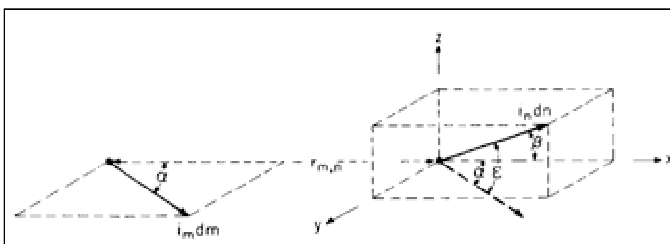


**Figure 1.** Angles of Ampère's force law.

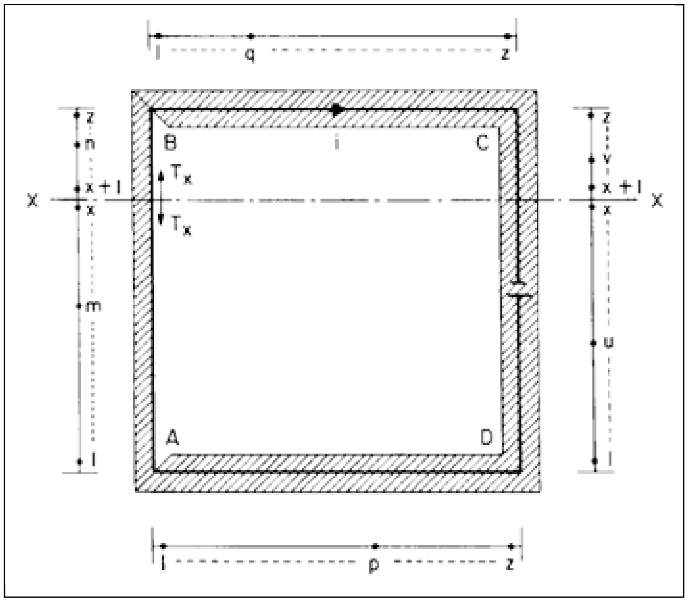

**Figure 2.** Square circuit with one free side.

equations, one for the force $\Delta F_m$ on the element $dm$, and the other for the force $\Delta F_n$ on the element $dn$. In vector form, with the previously employed notation, these two equations may be written

$$\Delta \vec{F}_m = (i_m i_n / r^2_{m,n})\ d\vec{m} \times (d\vec{n} \times \vec{1}_r)$$
$$\Delta \vec{F}_n = (i_m i_n / r^2_{m,n})\ d\vec{n} \times (d\vec{m} \times \vec{1}_r) \qquad (2)$$

where the direction of unit distance vector $\vec{1}_r$ is along the line connecting the elements and pointing toward the element at which the force is being determined.

In terms of modern field theory, the total force which an electric charge $e$ experiences in the presence, at its location, of an electric $E$-field and a magnetic $B$-field is

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B}) \qquad (3)$$

where $v$ is the relative velocity between the charge and the source of the $B$-field. The second term of (3) is generally called the Lorentz force and, with the help of the Biot-Savart law, it can easily be shown to be identical to the Grassmann interaction forces of (2).

The original papers and books on the Ampère-Neumann electrodynamics do not appear to have been translated into English. More than anything else, this explains the uncertainty which has arisen as to the measure of agreement between the Ampère and Lorentz forces on complete circuits and circuit portions. Many textbooks on electromagnetism written in the last 30 years do not mention the old electrodynamics. Some wrongly describe (2) as Ampère's force law. Most books today refer to an Ampère law which relates electric current to its magnetic field. It has nothing to do with Ampère's theory, which was based on action-at-a-distance and not fields.

In this confusion it is sometimes held that the two laws give mathematically identical results when applied to closed circuits. This is only a half-truth. They do agree on the vector sum reaction force between closed circuits, but disagree on the force distribution around the circuits. According to both laws the force on an isolated current-element due to a separate closed current is perpendicular to the element. The sharpest disagreement arises with regard to the force distribution around an isolated arbitrarily shaped circuit. The Lorentz forces are then still everywhere perpendicular to the circuit while the Ampère law now permits and requires the existence of longitudinal force components which give rise to the experimental phenomena of Ampère tension, electromagnetic jets,[8] and the recoil of railgun accelerators.[11] Because of integration singularities, the quantitative evaluation of longitudinal Ampère forces is entirely dependent on computer-aided finite current-element analysis. Therefore these forces could not have been fully appreciated prior to, say, 1960.

This paper presents a quantitative analysis of electromagnetically generated tension in straight conductors according to Ampère's empirical law. It goes on to discuss the technological consequences which should result from this tension. Later sections describe an experimental demonstration of the action of Ampère tension in aluminum wires.

**AMPERE TENSION**

When two current-elements lie on the same straight streamline we have in (1) $\cos \varepsilon = \cos \alpha = \cos \beta = 1$ and the mutual force becomes

$$\Delta F_{m,n} = i_m i_n\ (dm \cdot dn / r^2_{m,n}). \qquad (4)$$

Modeling his theory on Newtonian gravitation, Ampère assumed the force between two current-elements to act along the line connecting them. Then he arranged his formula (1) so that positive interaction forces would stand for repulsion and negative forces for attraction. Equation (4) cannot become negative and therefore invariably describes repulsion. In this way Ampère's electrodynamics leads to the remarkable prediction that long straight current-carrying metallic conductors should find themselves in tension unless the return circuit somehow cancels the tensile action.

Ampère *deduced* his law from a host of experimental data in the same way Newton arrived at the law of gravitation from astronomical observations. Ampère's summarizing paper,[2] entitled "Mathematical Theory of Electrodynamics Uniquely Deduced from Experiments," was republished as recently as 1958 and goes to great length to demonstrate the method of deduction. Therefore, Ampère claimed his four null experiments *proved* the existence of the longitudinal forces revealed by (4). For a subsequent confirmation of this claim he devised with de LaRive[6] an additional experiment which directly demonstrates the repulsion between conductor portions lying on the same straight line.

Accepting this position, there remains much to be done in the way of calculating the tensile forces in practical conductor arrangements and explaining why they remained hidden to generations of physicists who were not familiar with the Ampère-Neumann electrodynamics. Let us begin by considering a single filament of conductor elements. Since the currents in Ampère's law are of finite magnitude, this filament has to have a finite cross-sectional area and not be merely a line. Near the end of the twentieth century it would be absurd to pretend, as one might have at Ampère's time, that current-elements may be infinitely thin and infinitely short. The atomicity of conductor metals is likely to set definite limits to the subdivision of what must be considered the fundamental "particles" of Ampère's electrodynamics. In the absence of any other proposal we will assume that the atom, or unit lattice cell containing one atom, is the ultimate current-element. In this case the current-element would be about as wide as it is long. This aspect will be further discussed as the analysis of particular circuits proceeds.

An infinitely long straight conductor could be and has been treated as a closed circuit. Yet it would be futile to analyze it because, even with finite elements, the Ampère formula would give infinite tension at every point along this conductor. To prove anything about Ampère tension the investigation has to concentrate on closed metallic circuits of finite size. In a straight portion of a finite circuit the elemental repulsion indicated by (4) should create tensile stress. Could this stress be annulled by interactions with the remainder of the circuit? No generally valid answer can be given. However, if the tension can be shown to exist in a particular case there is reason to believe that it will also be found in other circuit geometries.

Consider the particular case illustrated by Figure 2 in which a square circuit carries a steady current $i$ and is adequately cooled to ensure constant temperatures. Sides $BC$, $CD$, and $AD$ of the circuit are firmly embedded in a dielectric structure which is rigidly anchored to the laboratory frame. $AB$ is a free length of wire resting against a wall meant to absorb the lateral force on $AB$.

Let $T_x/i^2$ be the specific tension in interatomic bonds across plane $X$ intersecting the wire $AB$. As further shown by Figure 2, each side of the square is assumed to be divided into $z$ equal-length elements thin enough so that the conductor may be treated as a single filament.

A major contribution to $T_x$ will come from the repulsion exerted by the general elements $m$ in $AX$ on the general elements $n$ in $XB$. Since (1) is independent of the unit of length we may choose this to be

$$dm = dn = 1 \text{ unit of length.} \tag{5}$$

With the labeling of current-elements indicated on Figure 2, the distance between the two general elements may be written

$$r_{m,n} = n - m. \tag{6}$$

The specific tension contribution by the $m$ - $n$ element combination is

$$T_1/i^2 = \sum_{m=1}^{x} \sum_{n=x+1}^{z} \{1/(n-m)^2\} \ . \tag{7}$$

this will be a maximum when $x = z/2$.

Next we consider the interactions of current-elements in $AB$ with other elements in sides $BC$ and $AD$. The interactions in question are all repulsions. This is due to the fact that the angle-function ($2 \cos \varepsilon - 3 \cos \alpha \cos \beta$) is negative for all relevant element combinations because $\cos \varepsilon = 0$ and $\cos \alpha \cos \beta = \cos \alpha \sin \alpha$ with $0 < \alpha < 90°$. Now we have to make some assumption about the mechanical behavior of the unsupported wire $AB$. It is very thin compared to its length and it will therefore have little strength as a strut, while being quite strong in resisting tension. We therefore treat it like an ideal string, recognizing that this must involve some approximation. Interactions between $BC$ and $BX$ are taken up by the tensile strength of the wire and do not exert tension on the atomic bonds across plane $X$. The same is true for interactions between $AD$ and $AX$. However, the repulsion between $BC$ and $AX$, as well as between $AD$ and $XB$, adds to $T_x$. This is due to $AX$ and $XB$ having no column strength. Hence by resolving the latter repulsions along $AB$ we obtain the second contribution to the specific tension across plane $X$, that is,

$$T_2/i^2 = \sum_{n=x+1}^{z} \sum_{p=1}^{z} (3/r_{p,m}^2) \cos^2 \alpha_n \sin \alpha_n + \sum_{m=1}^{x+1} \sum_{q=1}^{z} (3/r_{q,m}^2) \cos^2 \alpha_m \sin \alpha_m \tag{8}$$

where

$$r_{p,n}^2 = (n - 0.5)^2 + (p - 0.5)^2 \tag{9}$$

$$r_{q,m}^2 = (z - m + 0.5)^2 + (q - 0.5)^2 \tag{10}$$

$$\cos \alpha_n = (n - 0.5)/r_{p,n} \ ; \ \sin \alpha_n = (p - 0.5)/r_{p,n} \tag{11}$$

$$\cos \alpha_m = (z - m + 0.5)/r_{q,m} \ ; \ \sin \alpha_m = (q - 0.5)/r_{q,m} \ . \tag{12}$$

The 0.5 terms arise from the fact that the position of the current-element is a point halfway along its length.

The third contribution to $T_x$ derives from interactions between $AB$ and $CD$. The angle function for this side pair has everywhere $\cos \varepsilon = -1$ and $\cos \beta = -\cos \alpha$. Furthermore, since $\alpha$ varies from 45 to 135 deg, $2 \cos \varepsilon - 3 \cos \alpha \cos \beta = -2 + 3 \cos^2 \alpha$. This is never positive and then, because of the negative sign of (1), all interactions are again repulsions.

It is convenient to split $CD$ by the plane $X$ with general elements $u$ on one side and $v$ on the other. Symmetry ensures that every elemental repulsion with an upward longitudinal component is offset by a symmetrical interaction with a corresponding downward component. Therefore actions of $XC$ on $XB$ do not contribute to $T_x$. The same is true for actions of $DX$ on $AX$. However, tensile forces will be produced in $AB$ by the actions of $XC$ on $AX$ and by $DX$ on

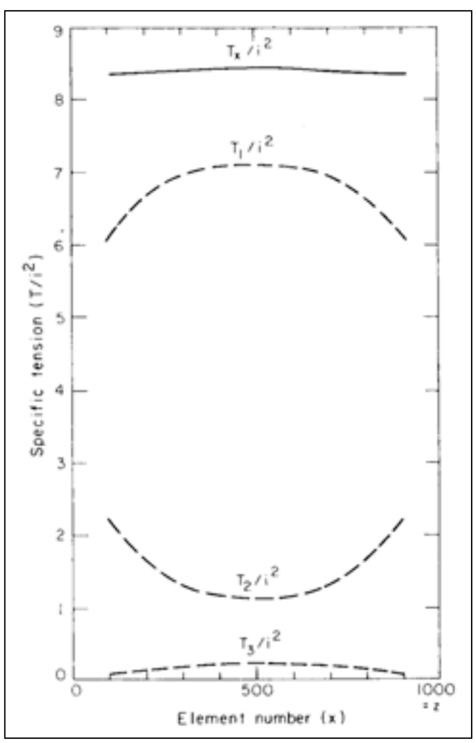

**Figure 3.** Specific tension in free side of square circuit.

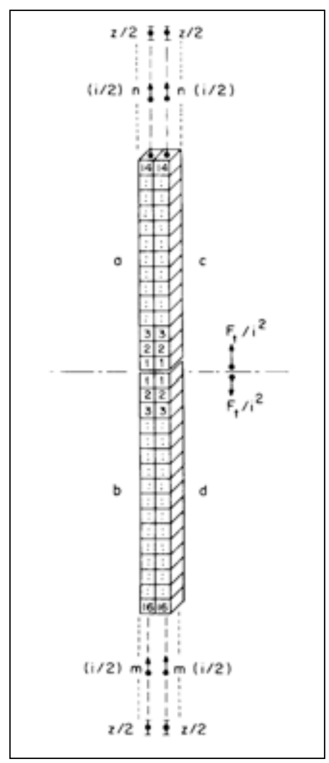

**Figure 4.** Tension across perpendicular midplane of two parallel straight filaments.

**Table 1.** Computer evaluation of (7) for *z* varying from 20 to 200 and $x = z/2$.

| $z$ | $T_1/i^2$ |
|---|---|
| 20 | 3.188 |
| 30 | 3.593 |
| 40 | 3.880 |
| 50 | 4.103 |
| 60 | 4.285 |
| 70 | 4.396 |
| 80 | 4.573 |
| 90 | 4.691 |
| 100 | 4.796 |
| 110 | 4.891 |
| 120 | 4.978 |
| 130 | 5.058 |
| 140 | 5.133 |
| 150 | 5.202 |
| 160 | 5.266 |
| 170 | 5.327 |
| 180 | 5.384 |
| 190 | 5.438 |

*XB*. They contribute

$$T_3/i^2 = \sum_{m=1}^{x} \sum_{v=x+1}^{z} (-1/r_{m,v}^2)(-2 + 3\cos^2 \alpha_v)\cos\alpha_v + \sum_{n=x+1}^{z} \sum_{u=1}^{x} (-1/r_{n,u}^2)(-2 + 3\cos^2 \alpha_u)\cos\alpha_u \qquad (13)$$

where

$$r_{m,v}^2 = (v - m)^2 + z^2 \qquad (14)$$

$$r_{n,u}^2 = (n - u)^2 + z^2 \qquad (15)$$

$$\cos\alpha_v = (v - m)/r_{m,v} \qquad (16)$$

$$\cos\alpha_u = (n - u)/r_{n,u}\,. \qquad (17)$$

The total specific tension in the wire *AB* may then be obtained by adding (7), (8), and (13):

$$T_x/i^2 = T_1/i^2 + T_2/i^2 + T_3/i^2. \qquad (18)$$

Figure 3 is a plot of the three tension components and their sum for $z = 1{,}000$. In the middle of side *AB* the tension is seen to be largely due to the repulsion of in-line current-elements. Near the ends of the side it is mostly produced by actions across the corners *A* and *B*. Side *CD* makes only a small contribution to the tension in *AB*.

It can easily be shown that the computed tension increases with *z*. At first sight this appears to be an unsatisfactory outcome of the Ampère electrodynamics. However, this difficulty can be overcome by making certain assumptions about the length-to-width ratio of the current-element. As a first step we calculate the most important tension contribution given by (7) across the mid-plane at $x = z/2$. Table 1 lists the results for *z* varying from 20 to 200. A regression analysis performed on this data revealed a very close fit to

$$T_1/i^2 = 0.19 + 1\text{n}\, z. \qquad (19)$$

It can be shown that the specific tension contributions $T_2/i^2$ and $T_3/i^2$ obey similar logarithmic laws. Hence $T_x/i^2$ will also be a logarithmic function of *z*. For $z = 1{,}000$, (19) gives the specific tension of 7.098 as compared to 7.099 obtained by finite element analysis. This extraordinarily good agreement gives us confidence to extrapolate (19) to much larger values of *z* which otherwise would have to be ascertained by a large computing expenditure.

Equation (19) tends to infinity with *z*. Hence if the current-element is assumed to be infinitely divisible, the Ampère electrodynamics becomes absurd. The same is true for an electrodynamics based on the Lorentz force law (2). We really have no choice but to accept finite size elements. Could the lower element size limit be determined by the distance between neighboring atoms? In metal lattices this would be of the order of $10^{-7}$ cm. It would amount to $10^9$ current-elements in *AB* of Figure 2, if the latter side is 100 cm long. Equation (19) then gives a specific tension of 20.91, which is only three times the tension obtained for $z = 1{,}000$. It is not an unreasonably large number and therefore lends some support to the idea that the atomic cell is the extent of the basic current-element.

It appears plausible that any reduction in current-element length from macroscopic to microscopic dimensions should be accompanied by a similar reduction in the cross-sectional dimension of the element. In other words, the specific tension of 20.91 probably applies to a conductor of $10^{-7}$ cm in diameter and 1 m long. For conductors of larger diameter, the atomic element concept requires the consideration of a bunch of parallel filaments, each being essentially a string of atoms.

What will be the tension in two adjacent strings of atoms which share the current of one absolute ampere? To obtain an answer consider the two square-section filaments of Figure 4. They have been subdivided into four portions *a*, *b*, *c*, and *d*. Each portion consists of $z/2$ cubic current-elements with their vectors all pointing in the same direction. The shape of a cube has been chosen for the convenience with which a solid conductor may be subdivided into cubic cells, and not because this is the atomic cell shape. Let us now determine the specific tension $T_1/i^2$ across the midplane of the filament combination when each filament carries half the total current, or $i/2$. The tensile force due to the interaction of portions *a* and *b* can be derived directly from (7) and (19). An equal component will arise from the interaction of portions *c* and *d*. Let these two components be $T_{a,b}$ and $T_{c,d}$, then

$$T_{a,b} = T_{c,d} = (1/4)(0.19 + 1\text{n}\, z)i^2. \qquad (20)$$

For the calculation of components $T_{a,d}$ and $T_{c,d}$ which, because of symmetry, are equal to each other, we find from Figure 4 that

$$r_{m,n}^2 = (\text{m} + n - 1)^2 + 1 \qquad (21)$$

$$\cos\varepsilon = 1 \qquad (22)$$

$$\cos\alpha = \cos\beta = (m + n - 1)/r_{m,n}\,. \qquad (23)$$

Applying Ampère's force law to portions *a* and *d* of the filament pair of Figure 4 and resolving the elemental interaction force in the direction of the current, we obtain

$$T_{a,d} = T_{c,b} = (1/4)i^2 \sum_{m=1}^{z/2} \sum_{n=1}^{z/2} (1/r_{m,n}^2) \cdot (2\cos\epsilon - 3\cos\alpha\cos\beta)\cos\alpha. \qquad (24)$$

Solving the simultaneous equations (21)-(24) by computer, and applying regression analysis to the results, revealed the logarithmic relationship

$$T_{a,d}/i^2 = T_{c,b}/i^2 = (1/4)\,(-1.64 + 1\text{n}\, z). \qquad (25)$$

Hence the tension $T_1$ across the midplane of the filament combination is

$$T_1 = 2T_{a,b} + 2T_{a,d} = (-0.73 + \ln z)i^2 \qquad (26)$$

which is smaller than the force given by (19). This result demonstrates that the amperian tension will be reduced if the current divides between two adjacent filaments. It is an important effect which will henceforth be called "longitudinal force dilution." Adding two more filaments to the arrangement of Figure 4, to make up a square-section conductor, would result in a specific tensile force of $(-1.23 + \ln z)$ which is distributed over the four filaments and thereby dilutes the tension even further.

In summary it has been shown that, for the circuit of Figure 2, Ampère's force law predicts the existence of tension in a current-carrying conductor, and that this tension is not predicted by the Lorentz force law (2). In the example of the square circuit a current of, say, 6,000 A = 600 ab-amp would create a tensile force of 2,385 g, if $z$ is taken to be 1,000. For an element length-to-diameter ratio of 1, this would be the tension in a 1 mm diameter wire of 100 cm straight length. The calculated tension would be sufficient to break an aluminum wire heated to 500°C, and it can therefore be verified or denied by experiment. There is good reason to believe that the same kind of finite element analysis would also reveal tension in rectangular and other circuit geometries. In reducing the current-element size from macroscopic to microscopic dimensions, the tension in an individual current filament appears to increase by as much as a factor of three. However, when the current is distributed over a number of closely bunched adjacent filaments, longitudinal force dilution takes place and this counteracts the apparent tension increase with element-size reduction. Elsewhere[11] it has been proved by measurement that macroscopic current-elements of the shape of cubes give results which are in good agreement with experiment.

**MACROSCOPIC CURRENT-ELEMENT ANALYSIS**

During the 80 years from 1820 to 1900, when the Ampère law was in wide use, current-elements were treated as being infinitely divisible. The result was a continuum theory which led to singularities in the integration of tensile forces because $r_{m,n}$ across the interface of conductor portions approached zero. This probably explains why so little was written in the nineteenth century about amperian tension in electric conductors.

Large current-elements of a cross-section equal to the whole conductor section have often been successfully employed to calculate the reaction forces between two complete circuits which were separated by at least 10 current-element lengths. It therefore seemed worthwhile to investigate if single filament representations of practical conductors can be helpful in estimating the magnitude of Ampère tension.

To do this a 100 x 40 cm rectangular circuit, made up of 0.25-in diameter copper rod and two liquid mercury links, was set up in a vertical plane. As shown in Figure 5, the uppermost side and 3 centimeters of each vertical leg were cut off and reconnected with liquid mercury contained in dielectric cups attached to the bottom portion of the circuit. The liquid metal gaps were less than 1 millimeter long. The electromagnetic lift force $F_l$ on the upper portion of the circuit was measured with a beam balance. As the results plotted on Figure 5 indicate, the reaction force was found to be proportional to the square of the current, giving a specific force of $F_l/i^2 = 10.30$. According to accepted pinch force theory,[12] the liquid mercury is responsible for a specific upward thrust of 1.00. Hence Ampère's force law should account for 9.30 of the specific force.

In the macroscopic current-element analysis of the lift force, the circuit was modeled as a single filament of 1 centimeter long elements, making perfect right-angled joints at each corner. Apart from computing the Ampère lift force, consisting partly of longitudinal components in the vertical legs and partly of transverse forces on the horizontal branch, the Lorentz force on the horizontal conductor was also computed by applying (2) to the 1 centimeter long current-elements. The results were as follows.

| | |
|---|---|
| Ampère: | $F_l/i^2 = 11.20$ (47 percent longitudinal) |
| Lorentz: | $F_l/i^2 = 11.24$ (all transverse) |
| Experiment: | $F_l/i^2 = 9.30$. |

Hence the single filament representation with current-elements of a length even longer than the conductor diameter

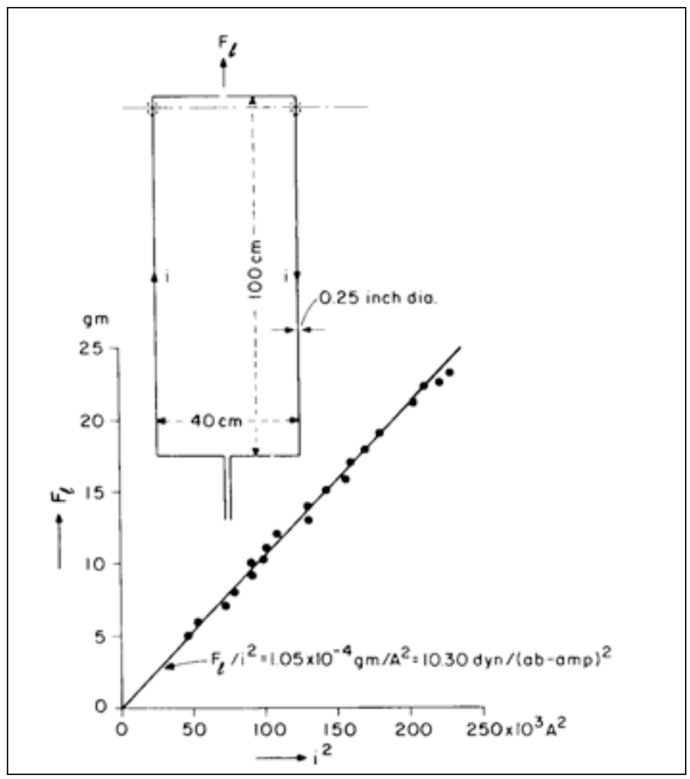


**Figure 5.** Lift force measurements on rectangular circuit.

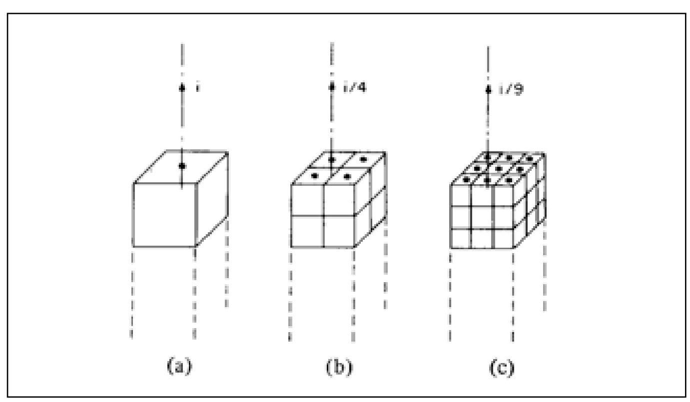


**Figure 6.** Cubic element subdivision of linear conductor.

gave a rough guide to the magnitude of the reaction forces between parts of the same circuit. In the chosen example the Ampère force appeared to overestimate the force by 20%. Part of this discrepancy may have been due to experimental errors.

The other interesting revelation was that, for the same element size, both Ampère's and the Lorentz force theory predicted almost identical tension forces. However in Ampère's electrodynamics 47% of the tension was due to longitudinal forces, while the Lorentz force was of course entirely a transverse force which produced tension indirectly. This coincidence and the prominence given to rectangular circuits in the past has no doubt contributed to the belief that Ampère's force law is not needed.

How should one proceed from here? The founders of the Ampère-Neumann electrodynamics were masters of analysis, but obviously did not succeed in finding an analytical solution for the directly induced conductor tension. Computer-aided finite element analysis was not available to scientists of the nineteenth and the first half of the twentieth century. It therefore is deemed desirable to pursue the latter technique a little further. The finite current-element has to be of a definite shape and the cube lies close at hand.

Consider an $a$ x $a$ square-section conductor of straight length $l$. If $l >> a$, the important midplane tension is then largely independent of any further increase in length. In the case of Figure 3, where $l/a = z = 1{,}000$, over 80% of the midplane tension is being contributed by the repulsion of in-line elements. Therefore, when dealing with very long straight conductors, we may ignore the return circuit and remember that this will underestimate the Ampère tension.

For $z = 10{,}000$ the specific midplane tension from (19) comes to 9.40. This would apply, for example, to a 100 m long conductor of one square-centimeter cross-section. Assuming the current to be 1,000 A = 100 ab-amp, the single filament model predicts a tension of only 96 g, or 0.096 kg/cm$^2$ tensile stress. It would produce a negligible amount of strain in spite of the high continuous current density of 1000 A/cm$^2$. It is therefore not surprising that Ampère tension has gone unnoticed in ordinary wires and cables used for the transmission and distribution of electrical energy.

The tensile stress will become more severe in fully loaded cryogenically cooled conductors. For instance, an aluminum conductor of the above dimensions held at the temperature of liquid nitrogen could possibly carry 10,000 A continuously, when it would be subject to 9.6 kg/cm$^2$ tensile stress. Even then the strain is quite modest.

Ampère tension is likely to be of critical importance in super-conductors. Type II superconducting filaments embedded in a copper matrix and cooled with superfluid liquid helium can support current densities of the order of 100,000 A/cm$^2$ over the combined copper and superconductor area. In the 100 m long one square-centimeter conductor this would give rise to 960 kg/cm$^2$ tensile stress which not only produces noticeable strain but would very likely change the superconducting properties of the rod, known to be strain sensitive.

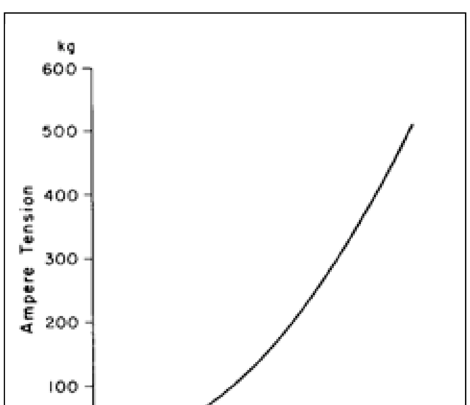

**Figure 7.** Plot of approximation (28).

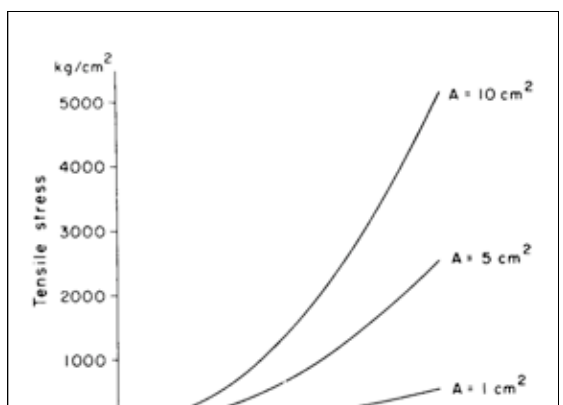

**Figure 8.** Conductor tensile stress according to (29).

The single filament representation of the linear conductor is the crudest model one can use. Finer subdivision of the conducting matter into smaller cubes should result in better approximations to the specific tension. Therefore, let every element of Figure 6(a) be subdivided into eight smaller cubes, as shown by (b). This multiplies the computational work by at least a factor of 64. Angles $\alpha$ and $\beta$ are then no longer zero for all relevant current-element combinations and (24) has to be used in addition to (7). To obtain a quantitative indication of the force dilution resulting from filament subdivision we analyze a relatively short conductor of 2 m length and 1 cm$^2$ cross-section. The return circuit would make a significant contribution to the maximum tension in this short conductor but we will not compute this. The midplane tension due only to the straight portion was found to be

single filament, Figure 6(a): $T_1/i^2 = 5.49$
four filaments, Figure 6(b): $T_1/i^2 = 4.71$
nine filaments, Figure 6(c): $T_1/i^2 = 4.55$.

This example demonstrates that the computed specific Ampère tension converges quite rapidly as the number of parallel filaments increases. Hence only a modest degree of subdivision will give good approximations. Although the ultimate current-element of the Ampère-Neumann electrodynamics is likely to be of atomic size, the usefulness of calculations involving macroscopic current-elements has now been demonstrated.

## PRACTICAL CONSEQUENCES OF AMPERIAN TENSION

*A. Normal Conductors*

It has been shown that the specific Ampère tension can approach a value of ten. For most practical purposes it would seem appropriate to assume that this tension will be at least

$$T \approx 5i^2 \text{ (dyn)} \qquad (27)$$

with $i$ expressed in absolute ampere. Using MKS units with the current $I$ in ampere, this approximation becomes

$$T \approx I^2/(20 \times 981{,}000) \text{ (kg)} . \qquad (28)$$

Approximation (28) has been plotted in Figure 7. For currents below 10 kA the tension is seen to be less than 5 kg. Few practical circuits carry more than 10 kA continuously. Furthermore, for steady currents of this order the size of the cross-sectional areas of copper and aluminum conductors are likely to vary between 50 and 100 $cm^2$, resulting in stresses of less than 100 $gm/cm^2$. They are negligible compared to thermally induced stresses created by Joule heating and other stresses caused by conductor supports.

Substantially higher currents may flow for a short time when power circuits are accidentally short-circuited. Fault currents of this nature may be as high as 50 kA(RMS) and under exceptional circumstances they may reach 100 kA. This means that, with the conservative approximations (27) and (28), power conductors could experience tensile impulses of the order of 100 to 500 kg lasting for a few cycles of the power frequency, until the fault current is interrupted by circuit breakers. It will be realized that ac short-circuit currents produce tension corresponding to the RMS current amplitude pulsating unidirectionally at twice the power frequency. The impulses appear strong enough to damage weak links such as conductor joints.

Many pulse current events should be dominated by longitudinal forces. Examples are the rupturing of fuses and the explosion of wires. Of particular interest, in this respect, is an observation made during many exploding wire experiments.[13] If capacitors are discharged through thin wires of a few inches length, the current will at first rise to a maximum and then decay to zero without discharging all the energy stored in the capacitors. After this interruption and dwell lasting for several microseconds, the current will start to flow again and completely discharge the capacitors. Photographic evidence proved that during the dwell period the wire is broken up in a number of pieces or liquid drops. Reignition of the explosion is brought about by an arc forming in the surrounding gas or vapor which envelops the wire pieces and usually evaporates all the metal. Currents in exploding wire experiments may reach 50 kA, which should result in tensile forces up to 100 kg.

It seems likely that the unexpected current pause observed in exploding wire experiments—which has never been satisfactorily explained—is the result of Ampère tension. The following sequence of events may be visualized. First the wire ruptures at some weak point as the temperature approaches the melting point. A gas arc immediately bridges the gap and restores tension. Further breaks develop which are all bridged by arcs. Every arc increases the voltage drop along the wire until this equals the applied voltage and extinguishes the current. Reignition of the discharge is another process not connected with Ampère tension. It could be caused by arc plasma diffusion which ultimately permits the striking of an all enveloping arc of low voltage drop.

*B. Superconductors*

Let $A$ ($cm^2$) be the cross-sectional area of a conductor carrying a uniformly distributed current density $j$ ($A/cm^2$). It then follows from (28) that the tensile stress over the conductor section is given by

$$\sigma = T/A \approx j^2 A/(20 \times 981{,}000) \ (kg/cm^2). \quad (29)$$

This stress has been plotted in Figure 8 for $A$ = 1, 5, and 10 $cm^2$ and current densities up to 100 $kA/cm^2$. Magnetic flux penetrates type II superconductors and, at least on a macroscopic scale, the current distributes itself approximately uniformly over the conductor cross-section. Hence, Figure 8 should apply to very long type II superconductors. If conductors of 10 square-centimeter cross-section could be loaded to 100 $kA/cm^2$, the amperian tensile stress would be formidable.

Since the advent of the CBS quantum theory of superconductivity it appears that quenching of the superconducting state at the critical current level is due to charge transport. However, for many years it was believed that the critical current $i_c$, which a straight type I superconductor could support, was the current which established the critical field $H_c$ at the surface of the conductor. This was known as Silsbee's rule, and it was largely borne out by experiment. If $r$ is the radius of the straight conductor, then the Silsbee rule states, in EMU

$$H_c = 2i_c/r. \quad (30)$$

Let the Ampère stress for this critical condition be $\sigma_c$, then from (27) and (30) it follows that

$$\sigma_c = T/A \approx 5i_c^2/(\pi r^2) \approx 1.25\pi H_c^2. \quad (31)$$

This last condition implies that the superconductor quenches at some critical tensile stress and the critical current as well as the critical field are measures of this stress. It is well-known that mechanical strain, resulting from stress, changes the critical current and the critical field strength. The question now arises, to what extent the critical parameters are affected by Ampère tension?

Experience has revealed that superconductors of 1 square-centimeter or more in cross-section will not sustain the current densities achievable with thin wires. The engineering

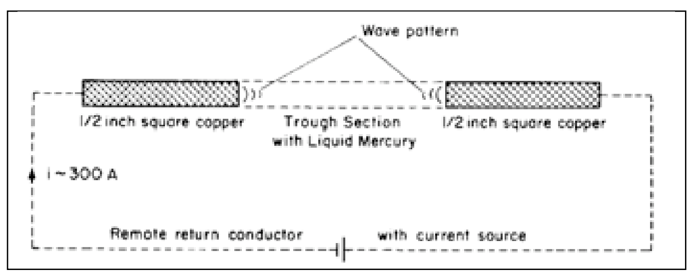


**Figure 9.** Generation of surface wave pattern at solid-liquid conductor interfaces.

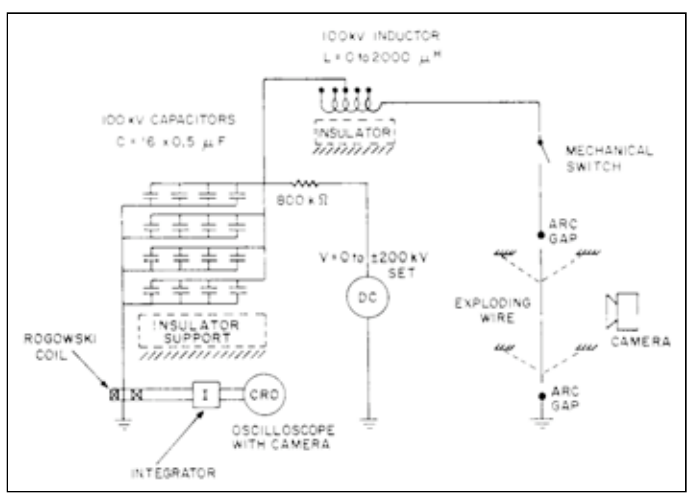


**Figure 10.** Exploding wire circuit.

solution of this problem has been to disperse large numbers of very thin superconducting filaments in a normal metal matrix and so decrease the effective current density over the combined cross-section of normal and superconducting metal. This practice also decreases the longitudinal stress in the composite conductor, which is likely to contribute to the improved performance.

In the first place the Ampère tension is felt by the superconducting filaments and from them it is transferred through the metallic bonding to the normal metal matrix. This mechanism should set up shear stress at the metal interfaces. When this stress exceeds certain limits, dissipative slip could result and generate heat which may also contribute to the quenching process. Frictional slip between parts of superconductive magnet windings is known to cause partial or complete quenching.

To further elucidate the shear action, let us take an example where 10,000 superconducting wires of 0.0254 cm diameter are embedded in a copper matrix to make up a composite conductor of 10 $cm^2$ cross-sectional area. When each wire carries 10 A, the total current is 500 kA, resulting in an average current density of 50 $kA/cm^2$. For this example Figure 8 gives a tensile stress of 1,250 $kg/cm^2$ and a total tension of 12,500 kg. This would set up a shear force of 15.7 kg/cm of periphery of the superconducting wires. It does not seem impossible that forces of this magnitude will break the bond between the two metals, at least at some of the weaker spots.

*C. Liquid Metal Conductors*

The simple experiment of Figure 9 is capable of showing an effect of longitudinal conductor forces in liquid mercury. The conductor dimension and the current magnitude are not critical for observing the effect. The experiment will work equally well with direct and alternating current.

The author used 0.5 $in^2$ copper bars set in a rectangular groove in a plastic board, part of the groove forming the liquid mercury trough. When about 300 A of current were switched on, an irregular wave pattern became apparent on the liquid mercury surface close to the interfaces with the copper rod. The waves disappeared almost instantaneously as the current was switched off. The disturbances were strongest right at the solid-liquid interfaces and they died out within a few centimeters of the interfaces.

Apart from a slight concave surface curvature on the mercury, due to surface tension of the liquid, the conductor was of uniform cross-section throughout. Transverse Lorentz forces should have pinched the liquid conductor equally all along its length and not create an irregular situation at the interfaces. No deformation of the mercury section due to pinching could be seen at the relatively small current of 300 A. This leads to the conclusion that the disturbances responsible for the observable surface wave pattern were caused by longitudinal Ampère forces.

Even when these facts have been recognized, it remains difficult to see how longitudinal forces can produce relative motion between some liquid metal atoms and others, as demonstrated by the wave pattern. Starting with the assumption that some atoms of the liquid metal are propelled away from the solid interfaces, it follows that others must return to take their places, or the liquid level would continue to fall at the interfaces, which it does not. For some of the atoms to return to the interfaces, not all can experience the same repulsion. In any case we should expect the tensile stress to be strongest in the center of the conductor and weakest along the corners. This will be better understood in conjunction with Figure 6(c). There the center filament has eight close neighbors which all contribute to the tension in the center. Each of the corner filaments has only three close neighbors and should feel a correspondingly lower tension. It would therefore not be unreasonable for liquid to flow away from the center of the interfaces and return to the periphery of them.

We still have to explain why, in the experiment of Figure 9, the wave motion and flow is strongest at the interfaces and disappears in the middle section of the liquid mercury trough. The following qualitative argument deals with this point. Each current-element is subject to two sets of forces. One set is generated by the far-actions of Ampère's force law. The other is due to the hydrodynamic push from neighboring elements caused by contact actions taking place throughout the liquid. In a solid conductor the contact action is taken up by the crystal lattice and it cannot be diminished by the conversion of some force into relative acceleration of atoms or current-elements. However, liquid conductor-elements may be accelerated relative to the body of the fluid. This reduces the local contact action by the product of mass times acceleration. The first liquid element at the solid interface will be backed up less strongly by contact action than it would be in a solid and accelerates into

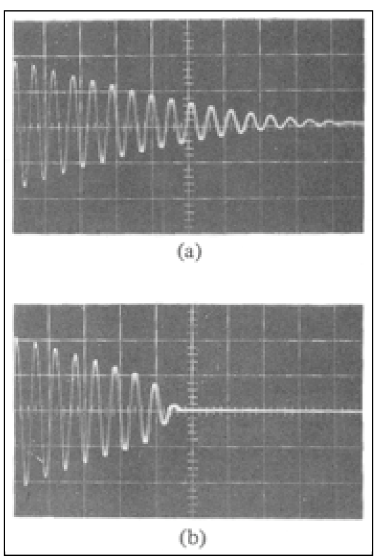


**Figure 11.** Capacitor discharge current oscillograms. (a) Without current-limiting arcs. (b) With current-limiting arcs.

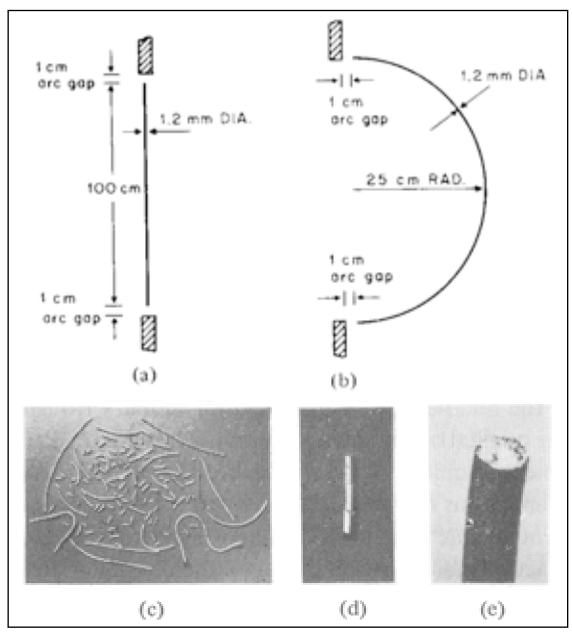


**Figure 12.** (a), (b) Straight and semicircular aluminum wires which are arc-connected to the discharge circuit. (c) Collection of wire fragments produced by discharge current. Distortion of fragments is due to hot pieces falling on laboratory floor. (d) Four short wire pieces spot welded together by arcs. (e) Appearance of brittle fracture face.

the body of the liquid. This effect tends to zero halfway between the two interfaces. It provides the explanation why no wave motion can be seen some distance away from the solid-liquid interfaces.

Another longitudinal force effect in a liquid metal conductor was discovered by Northrup.[12] His primary objective was to demonstrate the pinch effect on a horizontal liquid conductor. To make the effect more obvious he used a very light metal, that is a sodium-potassium alloy which remained liquid at room temperature and had about the density of water. With quite small currents he was able to produce deep and steep V-shaped depressions in a horizontal conductor of rectangular section. Moreover, the pinch was completely stable and it was of course attributed to the transverse Lorentz force. But Northrup also observed that liquid metal flowed rapidly *uphill* on the steep sides of the stable depression. This meant of course that liquid had to flow elsewhere to the bottom of the depression. The uphill flow was parallel to the tangential surface current in the metal and in every way compatible with the action of longitudinal Ampère forces. These two experimental observations suggest that longitudinal Ampère forces are likely to have a major influence on magneto-hydrodynamic phenomena in liquid metals.

## EXPERIMENTAL DEMONSTRATION OF THE ACTION OF AMPERE TENSION

### *A. Exploding Wire Circuits*

The circuit used at MIT for the experimental demonstration of current-induced tensile wire breaks is shown in Figure 10. It is typical of exploding wire circuits except for the large inductance and the relatively high voltage. The purpose of the inductance was to prolong the discharge. Higher voltages than normal had to be employed to overcome the inductive impedance and strike arcs to the test wire. It has been assumed that electric arcs in air have neither tensile nor longitudinal compressive strength and no forces can be transmitted through them to the exploding wire.

Most of the experiments were carried out with 1,000 μH circuit inductance and 8 μF capacitance. This resulted in a surge impedance of 11.2 Ω and required at least 56 kV to establish a current peak of 5 kA. Additional voltage was being dropped across the switching arc and the two connecting arcs to the test wire.

The circuit parameters caused the discharge current to oscillate at 1,830 Hz in the underdamped mode. When the capacitor bank was charged to 60 kV, the current would decay approximately exponentially, as shown in the oscillogram of Figure 11(a), without breaking the wire. It has been estimated that the 60 kV discharge was accompanied by a wire temperature rise of several hundred degrees of centigrade, which produced a thermal expansion of the order of 1%.

By subsequent increases of the discharge voltage in 2 kV steps, a pulse current level was reached at which the wire broke in one or more places. The hot fragments would fall to the laboratory floor and be distorted on impact. When repeating the experiment with a new wire and 2 kV additional voltage, the wire would break into a greater number of pieces. Finally, at 70 kV, the wire would show clear signs of melting which obliterated any evidence of tensile breaks. The oscillogram of Figure 11(b) indicates discharge current limiting and quenching due to the voltage drop along many series-connected arcs in air.

### *B. Straight Wires*

The most conclusive evidence for the existence of Ampère tension was obtained with straight wires mounted, as shown in Figure 12(a). In this arrangement all the Lorentz forces act perpendicularly to the wire axis and could not possibly produce tensile stress in the direction of the wiring axis. Yet the 1.19 mm diameter aluminum wire was shattered by brittle fracture into many pieces, as shown in the photographs of Figure 12. Electron micrographs, obtained with a scanning electron microscope, showed surface melting on the fracture faces to a depth of several micrometers, which is consistent with electric arcing across the fracture gap.

Longitudinal Ampère forces explain the brittle tension-breaks in the solid wire which was weakened by Joule heating. Can we think of any other mechanism which could explain this surprising phenomenon? Three come to mind. They are 1) separation due to pinch forces, and longitudinal fracture caused either by 2) acoustic vibrations or 3) thermal shock.

According to classical pinch force theory[12] the maximum pressure in a circular conductor occurs on the axis and is given by

$$P_{\max} = i^2/(\pi a^2),\ \text{dyn/cm}^2 \qquad (32)$$

where $i$ is the total conductor current in ab-amp and $a$ is the conductor radius in centimeter. The 99% pure aluminum wires of 1.19 mm diameter were subjected to peak pulse currents of at most 6,000 A = 600 ab-amp. The maximum pressure on the wire axis calculated from (32) comes to 33 kg/cm$^2$. Given sufficient time, pressures of the magnitude could extrude a hot wire. A careful micrometer survey of the fractured wire pieces revealed no evidence of any decrease in wire diameter. The extrusion forces existed only for a few milliseconds. This was too short a time interval for producing plastic deformation, let alone wire breaks.

The pinch force must have oscillated at 3,660 Hz, which is twice the discharge current frequency. This represents an acoustic vibration which in the long run could possibly lead to fatigue cracking and brittle fracture. However, the compressive stress in the outer regions of the wire is much less than along the axis and the vibrations could at most have lasted for 40 cycles. Pinch force-induced vibrations are therefore an unlikely cause of the wire breaks. Furthermore, any standing acoustic wave pattern should have resulted in wire pieces of equal length, which was not the case.

Nasilowski[14] was the first to observe how tensile breaks arrested a wire explosion. In one of his experiments he connected a longitudinal vibration detector to the wire to be exploded. He used a long dc current pulse which lasted for about 20 ms. Longitudinal vibrations and arcing started 10 ms after discharge initiation. It seems likely that the cause of the vibrations was tension relaxation of an initially taut wire when the first break occurred.

The electric arcs struck in the MIT experiments were of course accompanied by loud reports. Acoustic waves in air are not known to break wires even when they are due to lightning strokes. Hence, acoustic vibrations may be ruled out as the cause of tensile wire fractures.

Finally we consider thermal shock. Natural cooling of wires from red heat to ambient temperature, while the ends are free, is not known to have caused wires to break in tension. Convection and radiation cooling is a slow process and takes seconds if not minutes to be completed. Significant tensile stress could be set up by differential thermal expansion. Pronounced skin-effect heating might achieve this. However, the skin depth in aluminum at the ringing frequency of 1,830 Hz is 2 mm and more at elevated temperatures. This makes the current distribution over the 1.9 mm diameter wire almost uniform, giving no cause for thermal cracking.

The 100 cm long aluminum wire weighed three grams. According to Figure 3 the maximum specific tension was at least 7.0. For a peak current of 6,000 A this translates to a tension of 2.57 kg and a corresponding tensile stress of 231 $kg/cm^2$. It is equal to the ultimate strength of the material at around 300°C. The impact strength of the metal will be less. Therefore the first break in the wire could occur quite early in the discharge cycle provided the fracture pieces manage to separate in the available time. The repulsion between the two wire portions is equal to the tension just before the break. If the break occurs half way along the wire, it results in an acceleration of 2,570/1.5 = 1,713 times that of gravity. This produces a wire separation of $8 \times 10^{-3}$ cm in 0.1 ms, which seems adequate for a clean break. The mechanical isolation between the pieces must lower the tension in either portion and the current also decays with time. However, the reduction in strength with increasing temperature appears to permit further rupturing of the wire sections.

*C. Wire Semicircle*

The typical amperian mechanism of producing conductor tension is not only active in straight wires but also in curved sections. This will now be demonstrated with a wire semicircle which is arc-connected to the remainder of the discharge circuit. First we examine the mathematical situation in conjunction with Figure 13. Any contribution to the semicircle tension by the interaction of its elements with the remainder of the circuit will be ignored. This is likely to underestimate the Ampère tension, but the error over the middle portion of the wire will be quite small.

The semicircle *AXB* of Figure 13 is divided into *z* equal elements of arc, each subtending an angle of

$$\Delta\theta = \pi/z. \tag{33}$$

The elements along *XA* are labelled 1, 2, $\cdots$, *m*, $\cdots$, *x*; and those along *XB* are labelled 1, 2, $\cdots$, *n*, $\cdots$, (*z* - *x*). The distance between the two general elements *m* and *n* is denoted by $r_{m,n}$, and the arc *mXn* subtends the angle $\theta_{m,n}$. The angles of the Ampère force law obey the relationships

$$\alpha = \beta \tag{34}$$

$$\varepsilon = \theta_{m,n} = 2\alpha = 2\beta. \tag{35}$$

If *R* is the radius of the semicircle, then

$$dm = dn = (\pi/z)R \tag{36}$$

and

$$r_{m,n}^2 = 2R^2\,(1 - \cos\theta_{m,n})\,. \tag{37}$$

It can easily be shown that for any element combination on the semicircle the angle function of (1) is negative and therefore all Ampère interactions to be considered are repulsions. With (33)-(37) Ampère's force law may be written

$$\Delta F_{m,n}/i^2 = -(\pi/z)^2\,\{2\cos\varepsilon - 3\cos^2(\varepsilon/2)\}/(2 - 2\cos\varepsilon) \tag{38}$$

where $\varepsilon = (\pi/z)\,(m + n - 1)$.

The tangential component of this repulsion produces tension in a wire which is again assumed to behave like an ideal string with no bending strength. The transverse component of (38) tends to retain the shape of the semicircle and possibly accelerates the wire or its fragments away from the center 0. Noting that $\cos\alpha = \cos(\varepsilon/2)$, the elemental tension contribution may therefore be written

$$\Delta T_{m,n}/i^2 = (\Delta F_{m,n}/i^2)\cos(\varepsilon/2)\,. \tag{39}$$

Therefore the wire tension across the perpendicular plane containing 0*X* is given by

$$T_x/i^2 = \sum_{m=1}^{x}\sum_{n=1}^{z-x}(\Delta F_{m,n}/i^2)\cos(\varepsilon/2). \tag{40}$$

For the semicircle of Figure 12(b) *z* comes to 660 elements. Using this in the computer evaluation of (40) gave the results plotted on Figure 14. They show that the internally generated tension is relatively constant from end to end of the semicircle. Comparing this with Figure 3 we see that the tension in the semi-circle is expected to be of a similar mag-

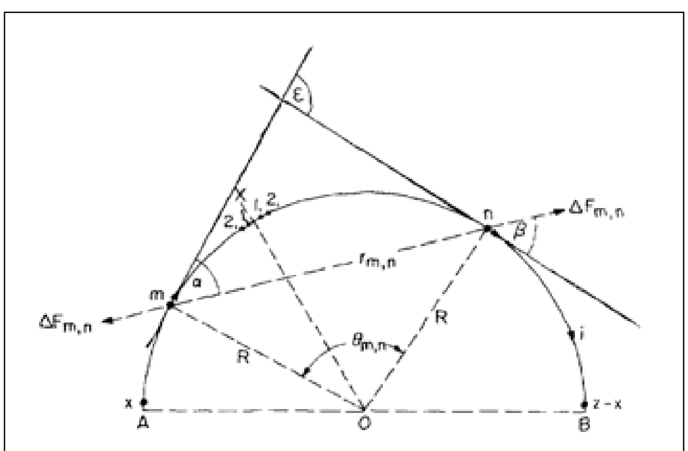


**Figure 13.** Construction for calculating Ampère tension in semicircle.

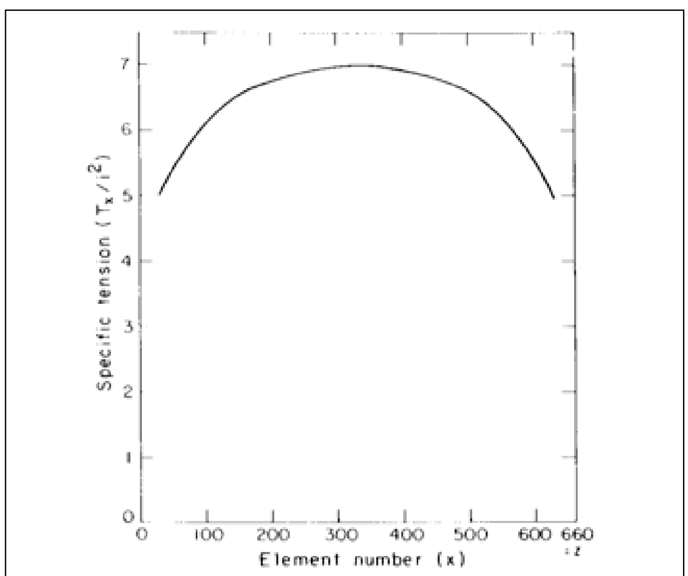


**Figure 14.** Computed specific Ampère tension in semicircle.

nitude as in the straight wire. Hence the same current pulses which fractured the straight wire of Figure 12(a) should also break the semicircle. Experiment fully confirmed this prediction of the Ampère force law.

## CONCLUSION

With the particular circuit of Figure 2 it has been proved that, under certain circumstances, the Ampère force law will predict tension in an electric conductor where it is not expected from the Lorentz force law. The disagreement has been put to the test by the experiment of Figure 12(a) and thereby resolved in favor of Ampère's law.

The fact that a component of the current-generated pondero-motive force does act in the direction of the current indicates that the metallic current-element is not merely a moving conduction electron, as hitherto assumed. A longitudinal force on the electron would accelerate or retard it without exerting any appreciable reaction on the metallic body. From this it may be concluded that the amperian current-element involves at least one lattice or liquid metal ion in addition to the conduction electron. The redefinition of the amperian current-element in terms of an electron-ion combination explains why the Ampère force law was found incapable of dealing with isolated charges in vacuum or gasses. It leaves the Lorentz force unchallenged in such important areas as particle accelerators, mass spectrometers, and electron optics.

While the element size is unimportant for the evaluation of interactions between separate circuits, it becomes the dominant factor when computing the force distribution around an isolated circuit. Ampère and his followers believed the current-element to be infinitely divisible. This created singularities in the integration of mechanical forces around individual circuits and so precluded the quantitative assessment of Ampère tension by analytical methods. The computer-aided finite current-element analysis outlined in this paper furnishes the required solutions. It also revealed that the amperian current-element *must* be of finite size or the Ampère-Neumann electrodynamics becomes absurd. Furthermore, it has been shown that the ultimate element may be of the size of an atom or unit atomic cell.

The resolution of conductors into quite large macroscopic current-elements gives approximate values of the Ampère tension which agree reasonably well with measurements. However, this agreement becomes unsatisfactory unless the length-to-width ratio of the element is about one. Macroscopic cubic current-elements have been found convenient and acceptable for computer analysis. Using cubic elements it has been shown that, at constant current, the Ampère tension will decrease with increasing conductor cross-section. This effect has been called "longitudinal force dilution."

The Ampère tension turns out to be quite small in air-cooled electric power conductors carrying slowly varying ac and dc currents. The much larger transient currents arising accidentally from lightning strokes and short-circuits of lines apply significant tensile stress to the copper and aluminum conductors widely used in power distribution. The practical consequences of Ampère tension are marked in pulse current devices, as for example railguns, exploding wires, and fuses. This tension is also expected to influence the design of large superconducting magnets for fusion and MHD generators.

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